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NOTE ON THE COMPUTATION OF THE ABSOLUTE PERTURBATIONS OF A COMET.

By Prof. Ormond Stone, University of Virginia.

The large eccentricities of the orbits of the periodic comets have thus far rendered it impracticable to compute tables of their motions as has been done in the case of planets and their satellites. Hansen, however, by the introduction of subsidiary quantities known as *partial anomalies*, has made a decided advance toward a solution of the problem. Since the publication of Hansen's Memoir, Gyldén and others have made some important modifications of his methods. The only applications of the method as yet made, so far as I am aware, have been to the partial computation of the perturbations of Encke's comet, and the results reached, though interesting and valuable, can by no means be considered satisfactory.

In Hansen's method the orbit is divided into two parts, the points of division being at equal distances on either side of the perihelion. The partial anomaly which includes the perihelion is made a function of the eccentric anomaly, while that containing the aphelion is made a function of the true anomaly. Afterward, by the application of special devices, the orbit may be divided into a greater number of parts.

The following is suggested as possibly offering some advantages:—

Put $\varepsilon = \eta_0 + \eta$, where η_0 is put successively equal to different values of the eccentric anomaly (ε) corresponding with points in the orbit selected arbitrarily In general, these will be selected so as to divide the orbit into equal portions with regard to ε. Put also $\sin \operatorname{am} z = \lambda(z)$, $\cos \operatorname{am} z = \mu(z)$, $\Delta \operatorname{am} z = \nu(z)$; then, within equal limits on each side of η_0, we may write

$$\begin{aligned} \sin \eta &= k\,\lambda(z), \\ \cos \eta &= \nu(z), \\ \frac{d\eta}{dz} &= k\,\mu(z), \\ k &= \text{modulus}. \end{aligned}$$

If, now, we put

$$\begin{aligned} l_{\prime} &= k \sin \varphi \sin \eta_0, & m_{\prime} &= -\sin \varphi \cos \eta_0, \\ l_{\prime\prime} &= -k \sin \eta_0, & m_{\prime\prime} &= \cos \eta_0, \\ l_{\prime\prime\prime} &= k \cos \varphi \cos \eta_0, & m_{\prime\prime\prime} &= \cos \varphi \sin \eta_0; \end{aligned}$$

we may readily obtain the following general equations:—

$$\frac{r}{a} = 1 + l_{\prime}\, \lambda(z) + m_{\prime}\, \nu(z),$$

$$\frac{r}{a} \cos f = l_{\prime\prime}\, \lambda(z) + m_{\prime\prime}\, \nu(z) - e,$$

$$\frac{r}{a} \sin f = l_{\prime\prime\prime}\, \lambda(z) + m_{\prime\prime\prime}\, \nu(z),$$

$$\frac{dg}{dz} = k \frac{r}{a} \mu(z);$$

where, as may be seen at a glance, $e = \sin \varphi$ is the eccentricity, a the semi major axis, r the radius vector, f the true anomaly, and g the mean anomaly.

If the intervals between the different values of η_0 be the same, k will be the same in all parts of the orbit, and a single development only will be needed of the quantities $\lambda(z)$, $\mu(z)$, and $\nu(z)$.

ON THE DESIGN OF STEPPED PULLEYS FOR LATHE GEARS.

By Prof. Wm. M. Thornton, University of Virginia.

The problem to be solved in this case is the determination of the diameters of a set of pairs of pulleys which will transmit the motion of the shaft to the lathe spindle with given angular velocity ratios by a belt of constant length. The fundamental formulæ for the solution of this problem are easily written down. We have

$$L = (\pi + 2\theta) R + (\pi - 2\theta) r + 2c \cos \theta,$$

$$\sin \theta = \frac{R - r}{c},$$

$$n = \frac{R}{r};$$